

The Cambridge Companion to
NEWTON

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1 Newton's philosophical analysis of space and time

INTRODUCTION: PHILOSOPHICAL CONTROVERSY OVER NEWTON'S IDEAS OF SPACE, TIME, AND MOTION

Newton's concepts of "absolute space," "absolute time," and "absolute motion" met with serious objections from such philosophical contemporaries as Huygens, Leibniz, and Berkeley. Among philosophers of the early twentieth century, after the advent of Special and General Relativity, the objections bordered on scorn: Newton's concepts were not only lately outmoded, but they were also epistemologically inherently defective, empirically unfounded – concepts not scientific at all, but "metaphysical," in so far as science is concerned precisely with "sensible measures" rather than obscure notions of what is "absolute." The prevailing idea was that Einstein had established not only a new theory of space and time, but a deeper philosophical viewpoint on space and time in general. From this viewpoint, space, time, and motion are essentially relative, and to call them absolute was an elementary philosophical error. As Einstein put it, General Relativity had taken from space and time "the last remnant of physical objectivity."¹

The philosophical motivation for this viewpoint seems obvious. Space cannot be observed; all that we can observe is the relative displacement of observable things. Therefore, if we observe two bodies in relative motion, to say that one of them is "really" moving, or that it is moving "relative to absolute space," is to pass beyond the bounds of empirical science. If we wish to decide which bodies are moving, we have to construct a frame of reference – that is, we must designate some reference-points to be fixed, and compare the motions of other bodies to these. Einstein held that any such choice of

a reference-frame is inherently arbitrary, and that a philosophically sound physics would be independent of such arbitrary choices; the "General Theory of Relativity" was supposed to be a theory in which all reference-frames are equivalent. To his philosophical followers, especially Hans Reichenbach and Moritz Schlick, Einstein was only saying what philosophers ought to have known, and a few had already suspected, on purely philosophical grounds. Contemporaries who had rejected Newton's views now seemed to have anticipated the eventual emergence of physics from its naive state.

In the 1960s and 1970s, however, many scientists and philosophers began to recognize what a few had known all along: that general relativity does not make space, time, and motion "generally relative," as Einstein had thought.² Instead, the theory postulates a spatio-temporal structure that is, in an obvious sense, just as "absolute" as the structures postulated by Newton. On the one hand, Einstein's field equation relates the geometry of space-time to the distribution of matter and energy. Thus, if "absolute" means "fixed and uniform," or "unaffected by material circumstances," then we can say that spacetime in general relativity is not "absolute," but "dynamical." On the other hand, spacetime in general relativity remains "absolute" in at least one philosophically decisive sense: it is not an abstraction from relations among material things, but a "physically objective" structure open to objective empirical investigation. Moreover, the theory does indeed make "absolute" distinctions among states of motion; it draws these distinctions in a way that departs dramatically from Newton's theory, but they remain physically objective distinctions that do not depend on the arbitrary choice of a reference-frame.

It became clear, then, that Newton's theory and Einstein's special and general theories all make essentially similar claims about the world: each specifies a certain "absolute" spatio-temporal structure, along with physical assumptions – primarily about the nature of force and inertia – that enable us to connect that structure with experience. In other words, conceptions of space and time are not arbitrary metaphysical hypotheses appended to otherwise empirical physics; they are assumptions implicit in the laws of physics. Defenders of Newton began to argue that "absolute" space-time structures are not so very different from other unobservable "theoretical entities" introduced into physics, such as fundamental particles

and fields. Accordingly, they ought to be judged by how well they function in explanations of observed phenomena. Any reasonable metaphysical question about space, time, and motion could thus be translated into a straightforward question about physics. For example, "is rotation absolute?" becomes, "does our best-established physical theory distinguish between absolute rotation and relative rotation?" and "is there an equally good or a better physical theory that dispenses with absolute rotation, or that refers only to relative motions?"³

From this point of view, we can ask of Newton's conceptions of absolute time, absolute space, absolute rotation, and absolute motion, "are they required by Newtonian physics?" And the answer is straightforward: Newton's laws presuppose absolute time, but not absolute space; they enable us to distinguish a truly rotating or accelerating body from one that is merely relatively rotating or accelerating; but they do not enable us to distinguish which bodies are "at rest in absolute space," or to determine the "absolute velocity" of any thing. Therefore Newton's laws require not absolute space, but a four-dimensional structure known as "Newtonian space-time." A straight line of this structure represents uniform motion in a straight line, and therefore its physical counterpart is the motion of a body not subject to forces.⁴ Einstein's theories postulate different space-time structures, based on different physical assumptions. Thus the theories should not be judged on purely philosophical grounds; it is, rather, a simple question of which theory is best supported by the empirical evidence. Had Newton said, "Spacetime is a four-dimensional affine space," instead of "Absolute space remains similar and immovable," there would have been no philosophical grounds for objection, but only (eventually) new developments in physics demanding new spacetime structures. Generally, on this point of view, our philosophical views about space and time should depend on our beliefs about physics.

Yet this seemingly simple approach to space and time has always been under philosophical suspicion. Einstein's chief objection had been anticipated by Leibniz: only the relative motions of bodies are observable, while space and time are not. How, then, could space, time, and motion be absolute? If we could construct a theory that made no reference to absolute space, time, and motion, ought we not to prefer it just for that reason? And even if "our best" physical

theory does make claims about space, time, and motion, do we not nonetheless have independent philosophical grounds to doubt their "absolute" status? For it seems absurd that any argument about observed spatial relations could prove that space itself is "absolute." Even to Newton's sympathizers, objections like these have always seemed challenging; to his opponents, they have seemed decisive. Hence whether motion is absolute or relative has appeared to be one of the perennial questions of philosophy.

As we will see, however, this approach to the philosophical questions of space and time is based on a fundamental misunderstanding of what Newton accomplished – indeed, a misunderstanding of the role that space and time play in physics. What it assumes is that what we *mean* by space, time, and motion, and what we mean by claiming that they are "absolute," is already established on purely philosophical grounds, so that we can then ask what physics has to say about these philosophical concepts. What it overlooks is that Newton was *not* taking any such meanings for granted, but *defining* new theoretical concepts within a framework of physical laws. Independently of such a framework, it is premature to ask, "did Newton successfully prove that space, time, and motion are absolute?" The proper questions are, what were Newton's *definitions* of "absolute space," "absolute time," and "absolute motion"? And, how do those definitions function in his physical theory?

NEWTON'S PHILOSOPHICAL CONTEXT

It was natural for Newton's contemporaries to misunderstand his purpose. Leibniz, for example, had an understanding of space, time, and motion, and of what it means to be a "substance" or to be "absolute," that arose from his own peculiar metaphysics. And to say that "space," "time," and "motion," as he understood them, are "absolute," rather than essentially relative, seemed to be an obvious mistake. But Newton explicitly proposed to ignore the prevailing philosophical uses of these terms, and to introduce theoretical notions of his own.

Although time, space, place, and motion are very familiar to everyone, it must be noted that these quantities are popularly conceived solely with reference to the objects of sense perception. And this is the source of certain

preconceptions; to eliminate them it is useful to distinguish these quantities into absolute and relative, true and apparent, mathematical and common.⁵

As Howard Stein first emphasized,⁶ the preconceptions that Newton had in mind were those of Descartes and his followers. Descartes had purported to prove that space is identical with extended substance. It followed that a vacuum is impossible, for wherever there is extension, there is, by definition, substance as well; it also followed that what we call motion "in space" is really motion relative to a fluid material plenum. From these foundations, Descartes developed a vortex theory of planetary motion: the rotation of the Sun creates a vortex in the interplanetary fluid, and the planets are thereby carried around in their orbits; similarly, the planets with satellites create smaller vortices of their own. Descartes would thus seem to have advanced a version of the Copernican theory, and attributed real motion to the Earth. But he equivocated on this point by his definition of "motion in the philosophical sense": while motion "in the vulgar sense" is "the action by which a body passes from one place to another," its motion "in the philosophical sense" is the body's "transference from the vicinity of those bodies contiguous to it to the vicinity of others."⁷ On this definition, Descartes could claim to hold both the heliostatic and geostatic views of the planetary system: the Earth is indeed revolving around the Sun in the vortex, but "in the philosophical sense" it is at rest, since it remains contiguous to the same particles of the fluid. Hence Descartes's assertion: "I deny the movement of the earth more carefully than Copernicus, and more truthfully than Tycho."⁸

Newton saw that such a definition is completely unsuitable for any *dynamical* analysis of motion, and in particular the dynamical understanding of the solar system. It implies that the choice between Copernicus or Kepler, on the one hand, and Ptolemy or Tycho, on the other, has nothing to do with the dynamical causes and effects of motion, but can only be made on the grounds of simplicity or convenience. From a certain philosophical point of view, of course, this is the desired conclusion. But the vortex theory itself – as advanced not only by Descartes, but by Leibniz and other "relativists" as well – assumed that the planetary system really is a dynamical system: that is, a system that is subject to the laws of motion, and whose parts are related by *causal interactions*. On that assumption, the fact that

planets orbit the sun, instead of moving uniformly in a straight line, requires some kind of causal explanation. Thus, Descartes's theory, *as a causal explanation* of the planetary motions, required a distinction between inertial motion and motion under the causal influence of a force. But this requirement is completely neglected by his definition of "motion in the philosophical sense." We begin to understand Newton's Scholium by properly understanding the question it addresses: what concepts of time, space, and motion are required by a dynamical theory of motion?

Asking this question about Newton's theory does not deny its connection with his profound metaphysical convictions – not only about space and time, but about God and his relationship to the natural world. On the contrary, it illuminates the nature of those convictions and their relationship to Newton's physics. For Newton, God and physical things alike were located in space and time. But space and time also formed a framework within which things act on one another, and their causal relations became intelligible through their spatio-temporal relations – above all, through their effects on each other's state of motion. The latter principle, which was implicit in seventeenth-century physics, was for Newton the link between physics and metaphysics: if physics is to understand the real causal connections in the world, then physics must define space, time, and motion so as to make those connections intelligible.

NEWTON'S DEFINITIONS

Newton begins by defining "absolute time" as time that, "without reference to anything external, flows uniformly."⁹ This means that, regardless of whether any particular mechanical or natural process flows equably – for example, regardless of whether the motion of any real clock or rotating planet really sweeps out equal angles in equal times – there is an objective fact, in "absolute time," about whether two intervals of time are truly equal. Absolute time also implies absolute simultaneity, so that each moment of time is defined everywhere, and it is an objective fact whether any two events happened at the same moment. These two principles define precisely what is presupposed about time in the subsequent arguments of the *Principia*. Newton's critics, however, have traditionally taken him to be asserting that "time is absolute," and that the meaning of such

a claim is established independently of physics. Leibniz, for example, assumed that if time is absolute, it must be (what he would call) a "substance," and so each moment must be a distinguished individual. This would mean that if the beginning of the universe were shifted from one to another moment of absolute time, some real difference would be made. But no such difference could be discernible; absolute time therefore violates the "Principle of the Identity of Indiscernibles," by which there cannot be two distinct things that do not differ discernibly. Therefore, to Leibniz, time cannot be "absolute," but can only be an "order of succession."

Yet in the notion of absolute time *as defined by Newton*, no such difference is implied. In fact, Newton explicitly rejects the idea that the moments of time (or space) have any identity above and beyond their mutual order and position, asserting (in strikingly "Leibnizian" terms) that "all things are placed in time with reference to order of succession; and in space with reference to order of position."¹⁰ The defining characteristic of absolute time is not the distinct individuality of its moments, but the *structure* of time, i.e., the fact that it flows equably and that equal intervals of time are objectively defined. The critical question is not whether Newton successfully proves that "time is absolute" – for this was never his purpose – but whether his definition of absolute time is a good one. And in the context of the *Principia*, this amounts to asking, does this definition have objective physical content? That is, can we define equal intervals of elapsed time without recourse to some arbitrary standard? Is there a good physical definition of what it means for time intervals to be equal, even if no actual clock measures such intervals exactly? The answer is "yes": this is precisely the definition of time implied by Newton's laws of motion, which postulate an objective distinction between inertial motions, which cross equal distances in equal times, and motions that are accelerated by an impressed force. In short, an ideal clock that keeps absolute time is simply an inertial clock: impossible to achieve in practice, but approachable to an arbitrary degree of approximation. Thus Newton's definition of absolute time is as well founded as his laws of motion. And this is why, in spite of all the traditional philosophical objections to it, it could only be overthrown by Einstein's introduction of new fundamental physical laws.

A similar analysis can be given of Newton's definitions of absolute space and motion. For Leibniz and others, to say that "space

is absolute" is to say that space is a substance, and thereby to attribute a distinct identity to each point of space. But if the locations of all things in space were shifted any distance in any direction, no real difference would be made; therefore (again by the Principle of the Identity of Indiscernibles), space cannot be absolute. Here again, however, in the definition of absolute space *given by Newton*, no such difference is implied. The defining characteristics of absolute space are that it remains "homogeneous and immovable," so that the parts of absolute space (the "absolute places") are truly at rest, and that translation from one to another absolute place is "absolute motion."¹¹ This means that there is a real difference between motion and rest in *the same* absolute place over time; but it does not imply any real difference between one universe, and another in which everything is shifted to a *different* absolute place; a body's state of motion depends on whether it remains in *the same* absolute place, but not on *which* absolute place it occupies. (Similarly, in Newtonian spacetime we can determine whether two velocities are the same, independently of their actual magnitude.) So Leibniz's classic arguments from the Principle of the Identity of Indiscernibles, cogent though they may be against a certain conception of space and time as "substances," are *not* arguments against the concepts Newton designated by "absolute time" and "absolute space."

Now, however, if we ask of absolute space what we asked of absolute time (is this a legitimate definition on physical grounds?) we encounter a problem. Unlike absolute time, absolute space entails a distinction that is not well defined according to Newton's laws: the distinction between rest and motion in absolute space. According to the laws of motion, a body moves uniformly in a straight line until an applied force causes it to accelerate, and the effect of the force is independent of the velocity of the body it acts upon. In other words, Newton's laws embody the principle of Galilean relativity, which Newton himself derived as Corollary 5 to the laws: "When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly straight forward without circular motion."¹² This means that nothing in the behavior of the solar system, for example, would enable us to determine whether it is at rest or moving inertially. Corollary 6 undermines absolute motion even further: "If bodies are moving in any way whatsoever with respect to one another and are

urged by equal accelerative forces along parallel lines, they will all continue to move with respect to one another in the same way as they would if they were not acted on by those forces."¹³ That is, nothing in the behavior of the solar system can even tell us whether the system is moving inertially, or being accelerated equally by some force from outside the system. Thus, though absolute space is invulnerable to the familiar criticisms from Leibniz, it is devastated by Newton's own concepts of force and inertia. Evidently this might have been otherwise: if the laws of physics measured force by velocity rather than acceleration, then dynamics could identify which bodies are truly at rest. Then we would have the physical definition of absolute space that Newtonian physics lacks. But in a Newtonian world, Newton's distinction between absolute motion and absolute rest cannot be realized.

That Newton was aware of this problem is clear from his discussion of absolute motion. He proposes to distinguish absolute from relative motion by its "properties, causes, and effects." And in the discussion of absolute translation, the properties can be simply defined: that bodies at rest are at rest relative to one another; that parts of a body partake of the motion of the whole; that whatever is contained in a given space shares the motion of that space. These properties together imply that we cannot determine the true state of rest or motion unless we refer motion to immovable space, rather than to some object or relative space that may be in motion. The latter properties, moreover, are directed against Descartes (without naming him, however). For they are not necessarily true of motion in Descartes's sense: if an apple moves, for example, the core remains at rest, as it is not moving relative to the skin that is contiguous to it. So Newton has given a more sensible analysis than Descartes of what we might mean by motion, assuming that we know which bodies are moving or resting in space. But that is precisely what we do *not* know: none of these properties enables us actually to determine empirically what a body's absolute motion is. An empirical distinction between absolute and relative motion first appears when we move from the properties of true motion to the causes and effects – causes and effects that have to do with inertia and force. And forces, as we have seen, can distinguish between acceleration and uniform motion, but not between "absolute motion" and "absolute rest."

The causes that distinguish absolute from relative motion are “the forces impressed upon bodies to generate motion.”¹⁴ Obviously, relative motion can be generated or changed without the action of any force, but true motion is only generated or changed by a force. By the same token, a body’s true motion necessarily “suffers some change” from the application of a force, whereas its relative motion need not: for example, if the reference-point by which we measure its relative motion is subject to the same force. Here a “relativist” might be tempted to ask, how does Newton know all of this about true motion? To ask this is to forget that Newton is elaborating the *definition* of true motion that is implicit in the principle of inertia. The critical question is, instead, does the definition define exactly what Newton wanted to define? Corollary 5 (or Corollary 6, for that matter) shows explicitly that it does not: the effects of impressed forces on the “true motions” of bodies are completely independent of the initial velocities of those bodies; therefore the causes of “true motion” provide a definition, not of motion with respect to absolute space, but of acceleration.

The same is true of the effects that distinguish absolute from relative motion: “the forces of receding from the axis of circular motion,” or centrifugal forces.¹⁵ “For in purely relative circular motion these forces are null, while in true and absolute circular motion, they are larger or smaller in proportion to the quantity of motion.”¹⁶ Such effects, even if we assume that they distinguish a true rotation from a relative motion, certainly cannot reveal whether a rotating body is at rest in absolute space. But what do they reveal? Newton discusses this in the most controversial part of the Scholium, the “water-bucket experiment.” The experiment is extremely simple: suspend a bucket of water by a rope, and turn the bucket in one direction until it is “strongly twisted”; then, turn the bucket in the contrary direction and let the rope untwist. As the bucket now rotates, the surface of the water will initially be flat, but relative to the bucket, it is rotating. By the friction of the rotating bucket, the water will gradually begin to rotate as well, eventually equaling the speed of the bucket, so that its motion relative to the bucket gradually ceases. Yet as the relative rotation of the water decreases, its “endeavor to recede from the axis of motion” – exhibited by the water’s climbing the sides of the bucket – increases correspondingly. The significance of this is plain. Newton is identifying the water’s

rotation by its dynamical effect, which is least when the motion in Descartes's sense is greatest, and greatest when the Cartesian motion is least.

Therefore, that endeavor does not depend on the change of position of the water with respect to surrounding bodies, and thus true circular motion cannot be determined by such changes of position. The truly circular motion of each revolving body is unique, corresponding to a unique endeavor as its proper and sufficient effect.¹⁷

Thus the Cartesian definition of motion ignores the very dynamical effects with which physics ought to be concerned. Newton explicitly points out, however, that his dynamical concept of motion is implicit in Descartes's own vortex theory. For in that theory,

the individual parts of the heavens [i.e. of the fluid vortex], and the planets that are relatively at rest in the heavens to which they belong, are truly in motion. For they change their positions relative to one another (which is not the case with things that are truly at rest), and as they are carried around together with the heavens, they participate in the motions of the heavens and, being parts of revolving wholes, endeavour to recede from the axes of those wholes.¹⁸

The true rotation of a body, then, cannot be judged from its motion relative to contiguous bodies, but only from the magnitude of the centrifugal effects it causes.

Critics of this argument have generally not defended the Cartesian view of motion against Newton's objections. But Newton was evidently trying to do more than distinguish true rotation from rotation in Descartes's "philosophical sense." This is clear from another thought-experiment: suppose that two globes, joined by a cord, revolve around their common center of gravity; suppose, further, that there are no other bodies, contiguous or otherwise, to which we can refer their motions. Even then, "the endeavor of the balls to recede from the axis of motion could be known from the tension of the cord, and thus the quantity of circular motion could be computed."¹⁹ In other words, the true rotation of a body is not only independent of its rotation relative to contiguous bodies; it is independent of *any* relative rotation. If Newton is correct, one could say of one body, in an otherwise empty universe, whether it is rotating or not.

This is the step that has always raised philosophical doubts: do the experiments prove that the water, or the pair of globes, is really

rotating? Could such an experiment possibly demonstrate the existence of absolute space? Is rotation relative to absolute space really the cause of the observed centrifugal forces? Perhaps the centrifugal forces on the water are not caused by motion relative to the bucket, but does this mean that they are independent of *any* relative motion, as the experiment of the globes purports to show? According to Ernst Mach, writing two hundred years after Newton, if Newton saw no need to refer motion to contiguous bodies, this is because he was tacitly referring all motion to the “fixed stars”. And even if we can deduce from Newton’s laws how bodies would behave in the absence of the fixed stars, we cannot deduce whether, in those circumstances, Newton’s laws would still hold anyway.²⁰

To Einstein, under Mach’s influence, Newton’s argument illustrated the inherent “epistemological defect” of Newtonian physics. Consider two spheres S_1 and S_2 , rotating relative to one another, and suppose that S_2 bulges at its equator; how do we explain this difference? Einstein says,

No answer can be admitted as epistemologically satisfactory, unless the reason given is an observable fact of experience... Newtonian mechanics does not give a satisfactory answer to this question. It pronounces as follows: The laws of mechanics apply to the space R_1 , in respect to which the body S_1 is at rest, but not to the space R_2 , in respect to which the body S_2 is at rest. But the privileged space R_1 ... is a merely factitious cause, and not a thing that can be observed.²¹

Einstein’s view became the “received view” of absolute rotation among philosophers of science. And even philosophers who have defended absolute rotation have accepted this challenge to show that absolute motion does provide a legitimate explanation.²² As our reading of Newton suggests, however, this critical view simply asks the wrong questions. Newton never claims to *prove* that the centrifugal forces on the water or the globes are caused by rotation relative to absolute space, or claims that any such experiment could demonstrate the existence of absolute space. What he says, instead, is that the centrifugal forces *define* absolute rotation. It makes no sense to ask, how does Newton know that S_2 is really rotating? S_2 is rotating *by definition* – more precisely, S_2 is rotating just because it satisfies the definition of absolute rotation. Thus Newton has not tried to *justify* the causal link between rotation and centrifugal effects,

but simply to identify it as definitive of true rotation. Thus he has defined a theoretical quantity, absolute rotation, by exhibiting how it is detected and measured by centrifugal effects. His discussion of the water-bucket makes this explicit: from the endeavor to recede from the axis, "one can find out *and measure* the true and absolute circular motion of the water, which here is the direct opposite of its relative motion" [emphasis added].²³ And concerning the globes, he states not only that from the tension on the cord "we might compute the quantity of their circular motions," but also that changes in the tension would provide a measure of the increase or decrease in rotation. "In this way both the quantity and the direction of this circular motion could be found in any immense vacuum, where nothing external or sensible existed with which the balls could be compared."²⁴ Again, we might think to ask how we really know that these effects provide a measure of absolute rotation, or by what right we can infer from such effects the quantity of absolute rotation. But this is as pointless as asking, by what right do we infer the magnitude and direction of an impressed force from the magnitude and direction of an acceleration? For this is just how Newton's laws define impressed force. In both cases, we are not inferring a theoretical entity from a phenomenon, but defining a phenomenon as the measure of a theoretical quantity.²⁵

Newton's argument, in sum, was never an argument from physical phenomena to metaphysical conclusions about the "absoluteness" of rotation. Instead, it was an argument of a sort that is fundamental to every empirical science: an argument that a novel theoretical concept has a well-defined empirical content. Like the definition of absolute time, and unlike the definition of absolute translation, the definition of absolute rotation does indeed have a basis in Newton's laws. And this means, again, that it is no less well founded than Newton's laws; if the universe in fact obeys those laws, we can always measure the true rotation of any body.

This interpretation of Newton's Scholium defies a long and continuing tradition, though its main point was already made by Stein in 1967.²⁶ But it is explicitly corroborated by Newton's other extended discussion of space, the manuscript "De gravitatione et aequipondio fluidorum."²⁷ For example, here Newton explicitly denies the conception of space and time as "substances" that provoked Leibniz's "indiscernibility" objection: "The parts of duration and space are

only understood to be the same as they really are because of their mutual order and position; nor do they have any hint of individuality apart from that order and position which consequently cannot be altered."²⁸ Newton concludes that space "has its own way of being, which fits neither substances nor accidents." He even suggests, for reasons not unlike those later given by George Berkeley, that the philosophical notion of "substance" is itself "unintelligible."²⁹

More important, "De gravitatione," much more explicitly than the Scholium, emphasizes that Newton's dynamical arguments concern the *definition* of true motion. His entire discussion of space and motion is contained in a "Note" to Definition 4: "Motion is change of place."³⁰ As Stein pointed out (1967), Newton begins immediately to justify this definition against "the Cartesians," by showing that Descartes's definition of motion is incompatible with the basic principles of mechanics. In particular, it is incompatible with the principle of inertia: if a body's true motion is defined relative to contiguous bodies, and the latter are the constantly flowing particles of the vortex, it will be impossible to define a definite path for the body. And in that case, it will be impossible to say whether that path is rectilinear or uniform. "On the contrary, there cannot be motion since there can be no motion without a certain velocity and determination."³¹

Newton also points out, however, that, alongside the "philosophical" conception of motion, Descartes makes casual or implicit use of a *physical* and *causal* conception of motion. For example, Descartes acknowledges that the revolution of a planet or comet around the sun creates centrifugal forces in the planet, a centrifugal tendency that must be balanced by the resistance of the fluid in the vortex. And this physical motion of the vortex itself is referred, not to "the ambient bodies," but to "generic" extension. Of course Descartes says that the latter is an abstraction from extended matter that exists only in thought; the vortical motion that produces the centrifugal forces is thus mere "motion in the vulgar sense," not true motion. But Newton observes that of these two parallel concepts of motion, it is the "vulgar" one, rather than the "philosophical" one, that Descartes appeals to in giving a *physical* and *causal* account of celestial motion. Therefore he argues that, of the possible ways of defining motion, we ought to choose that one that successfully defines a physical quantity, and that can therefore play a role in causal explanation: "And since the whirling of the comet around the Sun in his philosophical

sense does not cause a tendency to recede from the center, which a gyration in the vulgar sense can do, surely motion in the vulgar sense should be acknowledged, rather than the philosophical."³²

It might seem that Descartes's theory of motion is too easy a target, especially compared to a sophisticated account of the relativity of motion like that of Leibniz.³³ But Newton's objection to Descartes's definition is not merely its inadequacy or even incoherence, but also its inconsistency with dynamical principles that Descartes himself accepted. And this same objection applies to Leibniz: he appeals to a causal account of motion that is incompatible with his professed philosophical account. On philosophical grounds, as we have seen, Leibniz denies that there is a real distinction between one state of motion and another, and asserts the general "equivalence of hypotheses" about which bodies are at rest or in motion; consequently, he asserts that the Copernican and Ptolemaic systems are equivalent. Yet he very clearly does attach a *physical* meaning to the distinction between one state of motion and another. On the one hand, Leibniz presents a strange argument for the relativity of all motion. He claims to agree with Newton on "the equivalence of hypotheses in the case of rectilinear motions." But a curved motion is really made up of infinitesimal rectilinear motions, and so he concludes that a curved path is equivalent to a straight one, because they are equivalent in the mathematical sense that both are "locally straight." So all motions, rectilinear or curved, are equivalent.³⁴ On the other hand, according to Leibniz's own dynamical theory, the curved path is not *physically* – therefore not *causally* – equivalent to the straight path. This is because, on that theory, a body by its own inherent force can maintain its motion in a straight path, whereas a body cannot maintain a curved motion without the constant intervention of some other body. Indeed, the crux of his objection to Newtonian action at a distance is that it violates this principle:

If God wanted to cause a body to move free in the aether round about a certain fixed center, without any other creature acting upon it, I say it could not be done without a miracle, since it cannot be explained by the nature of bodies. For a free body naturally recedes from a curve in the tangent.³⁵

This passage establishes that Leibniz's understanding of rotation and centrifugal force was, at least in the context of physical explanation, the same as Newton's. And this is a natural consequence of Leibniz's

commitment to the vortex theory, in which the harmonic circulation of the planets results from a balance between their own "centrifugal tendencies" and the pressure of the ambient fluid.³⁶ More generally, such remarks reveal that, despite his "general law of equivalence," Leibniz's convictions about the fundamental nature of bodies, and their causal interactions with one another, depended on the concept of a privileged state of motion.

Leibniz's view exhibits the conflict, characteristic of seventeenth-century "relativist" views of space, time, and motion, between two opposing motives. On the one hand was the desire for a "relativistic" account of motion, in reaction against traditional Aristotelian objections to the motion of the earth. The classical argument was simply that terrestrial phenomena seem to reveal none of the expected effects of a rapid rotation or revolution; to accept the Copernican theory, one had to grasp the idea of "indistinguishable" states of motion, and to accept an "equivalence of hypotheses" about whether the earth is at rest. Only thus could Galileo argue that the terrestrial evidence is necessarily inconclusive, and appeal to the advantages of Copernicanism as an elegant account of celestial phenomena. On the other hand, the demise of Aristotle's theory of celestial motion – the "crystalline spheres" – produced the need for a causal account of motion, which would reveal the physical connections among the Sun and the planets. And the founding principle of that account, at least for Newton and Leibniz and their contemporaries, was Descartes's principle that the planets tend to travel in straight lines, but are forced by some physical cause into circulations around the sun. Leibniz maintained the mechanistic view that any such cause must act by immediate contact, while Newton accepted the possibility of "action at a distance," but, in any case, they shared the principle that a certain state of motion is "natural," and that any deviation from that state requires a causal explanation. Therefore, a "general law of equivalence" of states of motion would vitiate the very celestial mechanics that Leibniz and other Cartesians hoped to construct. If it made no *physical* difference whether the Sun orbited the Earth, or the Earth the Sun; if it made no physical difference whether the interplanetary medium were at rest, or rotating in a vortex; then there would be little hope of explaining the celestial motions by the physical interactions among the celestial bodies.

All of this shows that Newton's definition of absolute motion, in so far as it identifies the latter by its "causes and effects," is by no means an arbitrary definition, or an idiosyncratic one derived solely from his metaphysical views. Rather, Newton's definition identifies the very conception of motion that was implicit in seventeenth-century thinking about physical causes and physical explanations. His Scholium attempts (not entirely successfully, as we have seen) to characterize this conception precisely, and especially to separate it from philosophical "preconceptions" about relativity that are irrelevant to the task of physical explanation. In other words, instead of a metaphysical hypothesis to account for dynamical effects, Newton has offered a conceptual analysis of what is presupposed about motion – by Descartes, Leibniz, and every other seventeenth-century mechanist – in ordinary reasoning from motion to its physical cause.

THE SYSTEM OF THE WORLD

The Newtonian conception of motion has an obvious yet remarkable consequence: whether the planetary system is geocentric or heliocentric can no longer be settled by adopting the simplest hypothesis, but is now a straightforward empirical question. For, assuming the laws of motion, Book 3 of Newton's *Principia* argues from the celestial motions to the physical forces that cause them. Again, any post-Cartesian physicist would infer, from the fact that a planet travels in a closed orbit rather than a straight line, that *some* force keeps it from following the tangent; Newton, drawing on the work of Galileo, Huygens, and others, reasoned mathematically from the precise characteristics of the orbit to the precise characteristics of the force. And this reasoning leads eventually from Kepler's laws of planetary motion to universal gravitation.³⁷

Throughout this reasoning from motions to forces, Newton remains neutral between the geocentric and heliocentric theories. Once the forces are known, however, we can compare the masses of the celestial bodies by comparing the forces they exert on their satellites. From there, a very simple argument determines the physical center of the system. First, suppose (Hypothesis 1) that the center of the system (whatever it is) is at rest.³⁸ "No one doubts this, although some argue that the earth, others that the sun, is at rest in

the center of the system." Then (Proposition 11) the common center of gravity of the system must be at rest. For by Corollary 4 to the laws of motion, "that center either will be at rest or move uniformly straight forward. But if that center always moves forward, the center of the universe will also move, contrary to the hypothesis." The conclusion is immediate: "Proposition 12: That the sun is engaged in continual motion but never recedes far from the common center of gravity of all the planets."³⁹ In other words, if the planetary system is a dynamical system, whose members interact according to the accepted dynamical laws, then no body is at rest, for, by the third law of motion, to every action of every body there is an equal and opposite reaction, and only the center of gravity of the system can remain at rest. However, the comparison of masses reveals that most of the mass of the system is contained in the sun. Therefore, "if that body toward which other bodies gravitate most had to be placed in the center . . . that privilege would have to be conceded to the sun."⁴⁰

Newton's argument is that, given the laws of motion and the observed behavior of the planets and the sun, we can infer their causal influences on one another and their relative masses; when all of this is known, the structure and motion of the system – "the frame of the system of the world" – is determined. But, as Newton well knew, the system is determined only up to a point. By Corollary 5, no dynamical analysis of the solar system can reveal whether the system as a whole is at rest or in uniform motion. And Corollary 6 renders the analysis still less determinate. But none of this affects Newton's dynamical analysis:

It may be alleged that the sun and planets are impelled by some other force equally and in the direction of parallel lines; but by such a force (by Cor. vi of the Laws of Motion) no change would happen in the situation of the planets to one another, nor any sensible effect follow; but our business is with the causes of sensible effects. Let us, therefore, neglect every such force as imaginary and precarious, and of no use in the phenomena of the heavens.⁴¹

The causal analysis of the motions within the solar system establishes a close approximation to Kepler's heliocentric system, whatever the motion of the system as a whole. And the geocentric theory is revealed to be physically impossible, precisely as it would be

physically impossible for a baby to whirl a large adult around its head on a string: in both cases the smaller body must revolve further from the center of gravity.

Philosophically this argument is not very different from the Leibnizian argument for a heliocentric vortex. The latter, too, reasons from accelerated motions to their physical causes, and it infers from the nature and magnitude of the Sun that it, rather than the Earth, has the required causal efficacy to serve as the physical center of the system. Therefore, on Leibniz's physical theory as well as on Newton's, whether Ptolemy or Copernicus was more nearly right is a physically meaningful question. It should be emphasized, moreover, that the same comparison can be made between Newton's theory and general relativity. Philosophers used to say that general relativity had finally established the equivalence of the Copernican and Ptolemaic systems, except to the extent that one might be "simpler" than the other.⁴² Precisely as in Newton's theory, however, in general relativity the planetary orbits are determined by the mass of the Sun. The mass causes spacetime curvature, instead of a gravitational field in Newton's sense, but there remains an essential similarity: the mass required to account for the precise curvature of the planetary orbits is the same in both theories, and on either theory the Earth's mass is too small. So the two systems are, on physical grounds, as *inequivalent* in Einstein's theory as they are in Newton's. The decision between them is not an arbitrary choice of reference-frame, but the outcome of a dynamical analysis, based on the principle that states of motion can have genuine dynamical differences.

CONCLUSION: AN EMPIRICIST VIEW OF SPACE, TIME, AND MOTION

Newton's conceptions of space, time, and motion were long regarded as metaphysical ideas whose place in empirical science was open to dispute. Now we can finally see that they were, instead, exemplary of the way in which science gives empirical meaning to theoretical notions. A spatio-temporal concept belongs in physics just in case it is defined by physical laws that explain how it is to be applied, and how the associated quantity is to be measured; Newton called

"absolute" precisely those quantities that could be so defined. By this standard, absolute space does not belong in Newtonian physics, since absolute translation in space is not a physically measurable quantity. But absolute time, absolute acceleration, and absolute rotation are well-defined concepts that are, as we saw, implicit in classical thinking about physical causes. Thus philosophical questions about these concepts could become empirical questions. In particular, the question of what is really moving in the solar system was reduced to simple empirical questions. Which bodies exhibit the dynamical effects that are definitive of true rotation? Where is the center of gravity of the system, and what body is closest to that center?

The controversy over this theory of motion can be compared to the controversy over Newton's theory of gravitation as an action at a distance. To his scientific and philosophical contemporaries, action at a distance contradicted the very concept of physical action, which was supposed to be possible only by direct contact. But for Newton, action is defined by the laws of motion, which provide empirical criteria for measuring the action of one thing on another; if the planets and the sun satisfy these criteria in their direct mutual relations, then they are acting on one another. Thus the question of action at a distance became an empirical question. We can also compare this to the controversy over non-Euclidean geometry in the nineteenth century. Many philosophers found it inconceivable that space could possibly be curved; this seemed contrary to the very concept of space.⁴³ According to Gauss, Riemann, and Helmholtz, however, when we make precise the empirical meaning of the claim that space is curved, we see that it is no more contradictory than the claim that space is not curved. Both claims derive their meaning from physical assumptions about the behavior of bodies and light – for example, that "light rays travel in straight lines"; just this understanding of the meaning of curvature makes it an empirically measurable quantity, and makes the question whether space is curved an empirical question. Similarly, Newton showed that the familiar assumptions about inertia and force – specifically, that "bodies not subject to forces travel uniformly in straight lines" – suffice to define acceleration and rotation as empirically measurable quantities. His critics insisted that, to be an empiricist about space and time, one had to define motion as change of relative position; Newton's philosophical insight was that